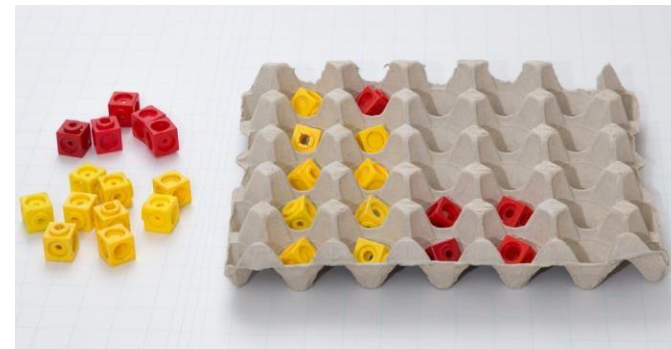
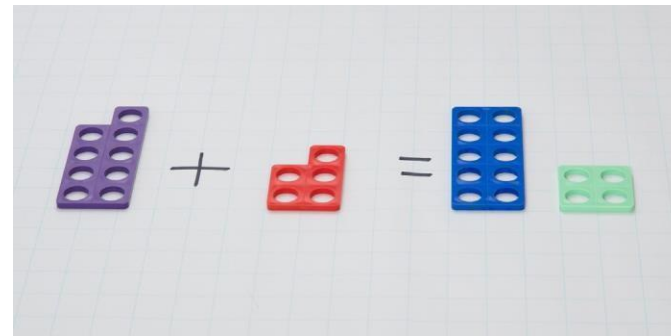
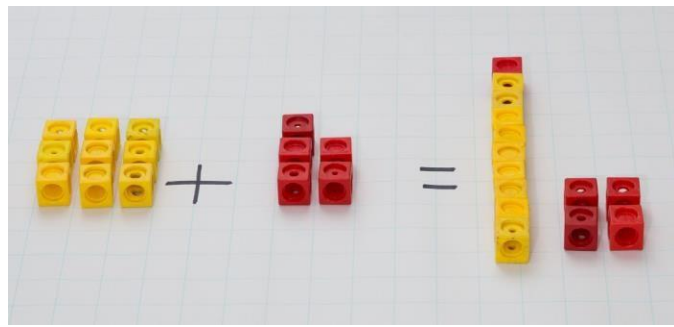




# Forest Hill Partnership

## Calculation Policy



### Calculation Policy – Forest Hill Partnership

What is this Policy for?

This policy is intended to demonstrate how to teach different forms of calculation at Staple Hill Primary. It is organised by stages to show a progression through each of the four operations. This policy is designed to help teachers at the school ensure that calculation is taught consistently in all year groups and to aid them in helping children who may need support or further challenge.

The policy is also designed to help parents, carers and other family members support children's learning in line with the teaching at school.

How to use this policy?

Each page follows the same format to help find the information needed. At the Forest Hill Partnership schools, we teach mathematics using the CPA approach – concrete, pictorial, abstract. This three step way of learning is based on many years of research and educational success. When learning something new, we always move through the three steps:

- Step one: C = **concrete** - children start to learn by using real objects or manipulatives to perform the ‘real’ story.
- Step two: P = **pictorial** - children draw the objects as they are at first and then move to a representation. This supports children in embedding the ‘real’ story of what they are doing.
- Step three: A = **abstract** - this is the written sum or number sentence or algorithm we call this the ‘maths’ story.

Children will always work through the steps in the CPA order with the aim that the children can quickly move to using just the abstract ‘Maths’ story as soon as they are ready. Different children will move through this process at varied speeds. Some children may only need to use the Concrete and Pictorial a couple of times before they are secure enough to move to using only the Abstract. Other children may require all three steps for an extended period.

Our policy is set out to map each stage of calculation across the three steps of learning. It contains additional information as well as photographs. Key language is highlighted in red in the first column.

Equipment/resources:

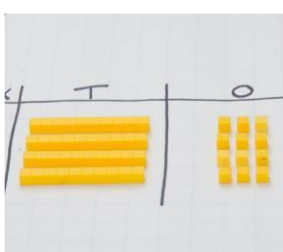
We use a range of concrete manipulatives and pupils should use a range when exploring mathematics. In any of the examples shown the concrete resource can be replaced by a different resource of a child’s choice.



Real life objects



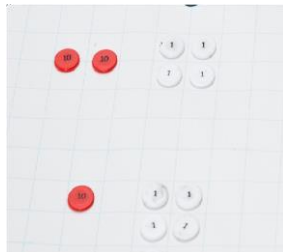
Unifix cubes Numicon counters beads



Number/counting



BaseTen/Dienes



Number line Place value

Vocabulary

Children should be encouraged to use a wide range of mathematical vocabulary to discuss the ‘maths’ using high levels of oracy. This is supported by the NCETM mathematical glossary. Children will have these discussions modelled by the teacher and will be provided with a scaffold using sentence stems in every lesson. We also use certain consistent definitions that will ensure that we teach maths in a consistent way throughout the school.


**Addition:** addend, sum/total. Part and Whole

**Subtraction:** subtrahend, minuend,

difference**Multiplication:**

**Division:** dividend, divisor, quotient

**Fractions:** whole and part. Denominator (written first) and numerator



$$\frac{3}{4}$$


*Numerator*

*Denominator*

Addition:

$$8 + 3 = 11$$

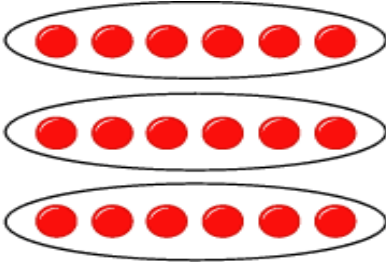
Addend Addend Sum or Total



factor factor product

$3 \times 6 = 18$

number of groups number in each group number in all



dividend divisor quotient

$18 \div 3 = 6$

number in all number of groups number in each group

Subtraction:

$$8 - 3 = 5$$

Minuend Subtrahend Difference

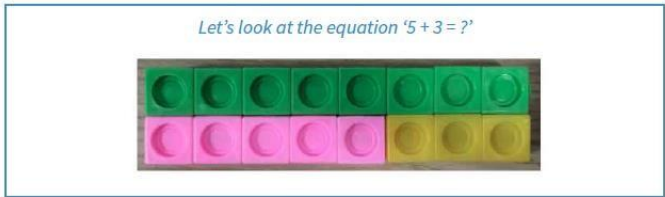
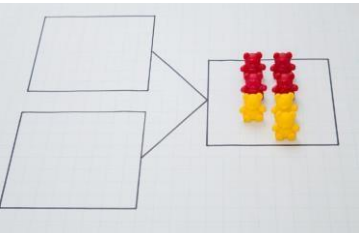
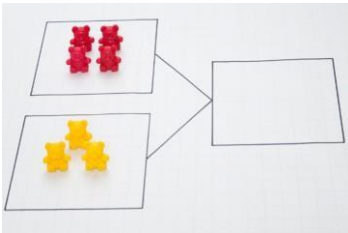
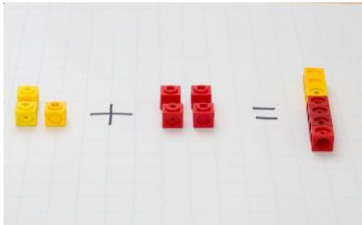
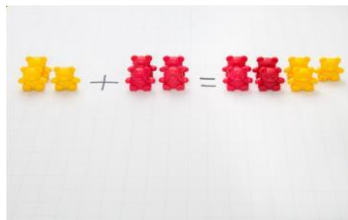
# Addition

## Objective

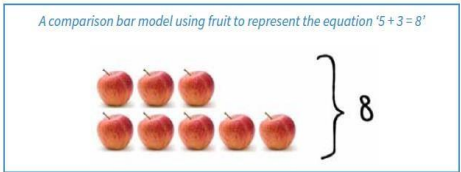
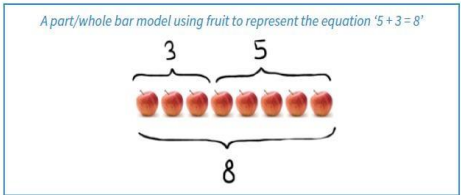
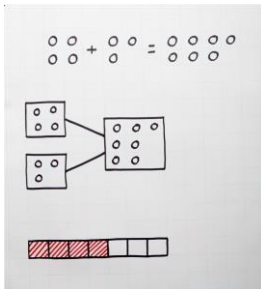
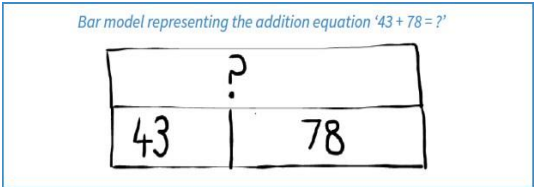
Stage 1: Combining two parts to make a whole.

more  
altogether  
add  
plus  
equals  
total  
make

## Concrete

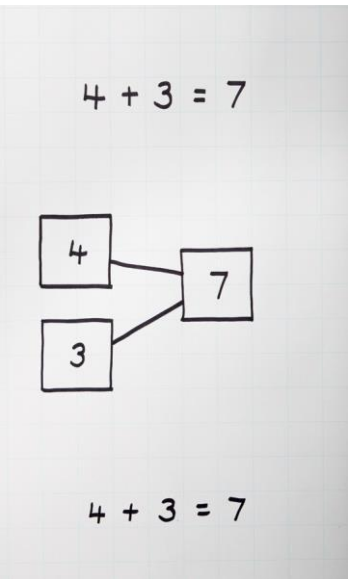


## Pictorial



Children should begin by drawing the real life object then move to an abstract model to represent each item  
e.g. a cube or counter

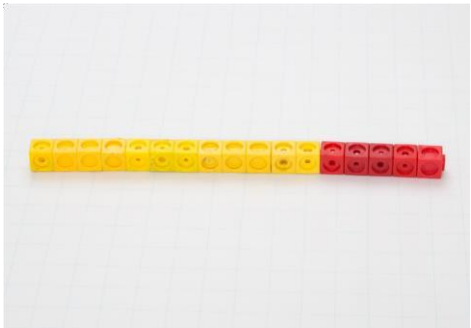
## Abstract



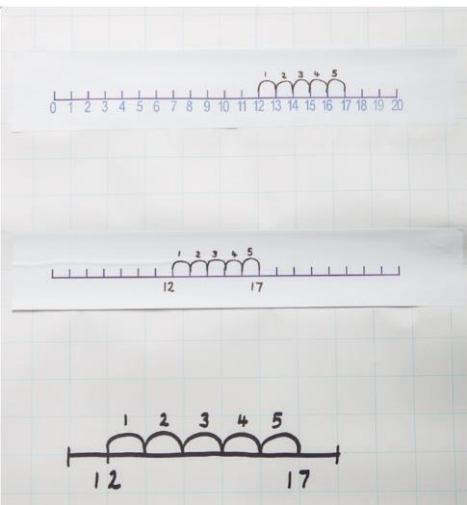
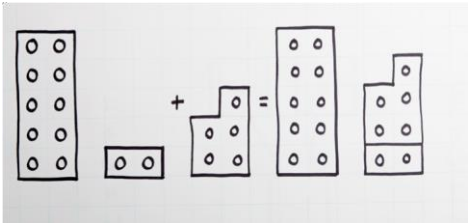
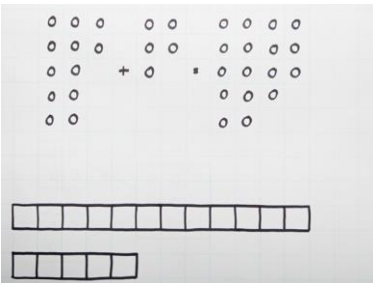
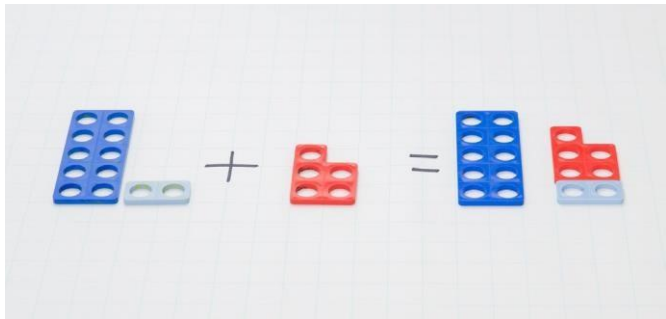
Use the part-whole diagram first then move to the abstract number sentence.

Stage 2:  
Start at the bigger  
number and count  
on.

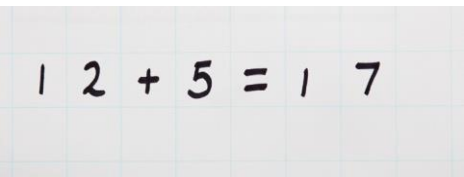
bigger  
smaller  
count-on  
and  
tens  
ones



Start at the larger number and then count on the smaller number 1-by-1 to find the answer.



Number line:  
children should start with a line with all the numbers included, then move to a line without the numbers where children decide which numbers to include. Finally moving to children drawing their own.



Children should hold the larger number in their head and count on the smaller number to find the answer.



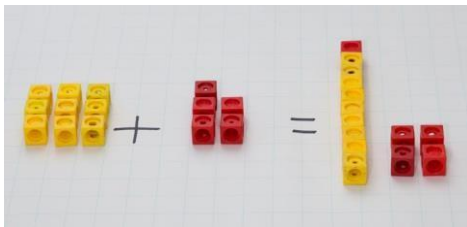
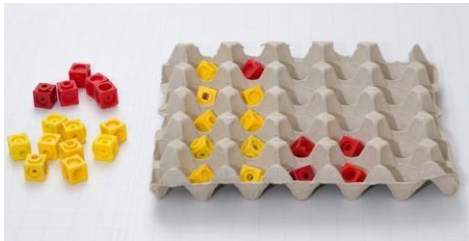
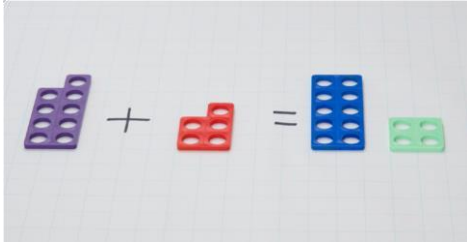
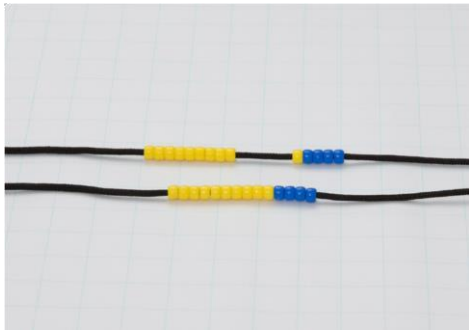
# Addition

## Objective

Stage 3: Regrouping to make 10

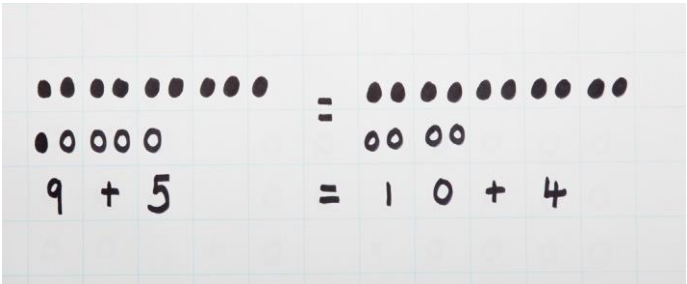
regroup  
number bonds  
number facts

## Concrete



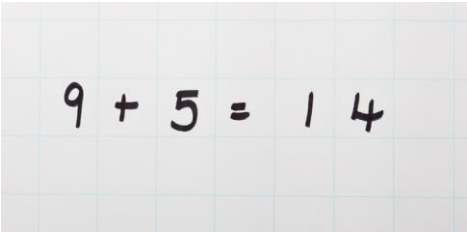
Start with the bigger number and use the smaller number to make a 10 first.

## Pictorial



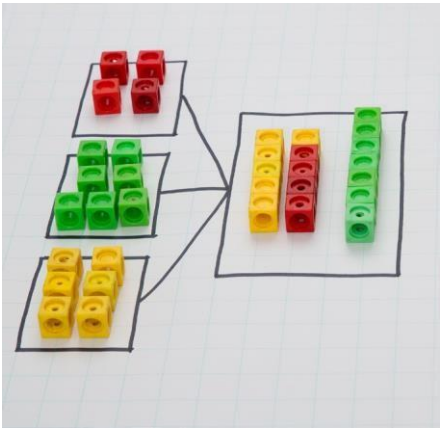
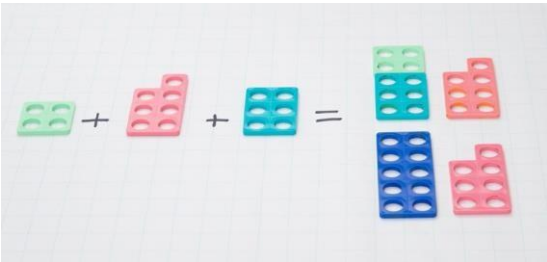
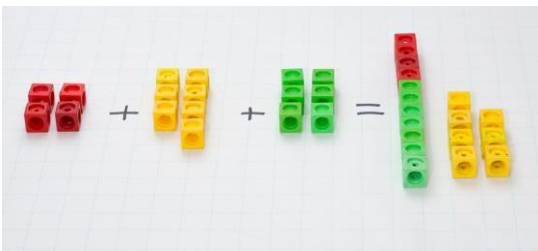
Use pictures of the object or a pictorial representation  
e.g a circle or a square

## Abstract

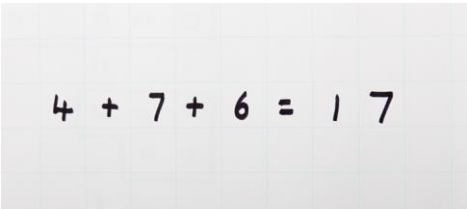
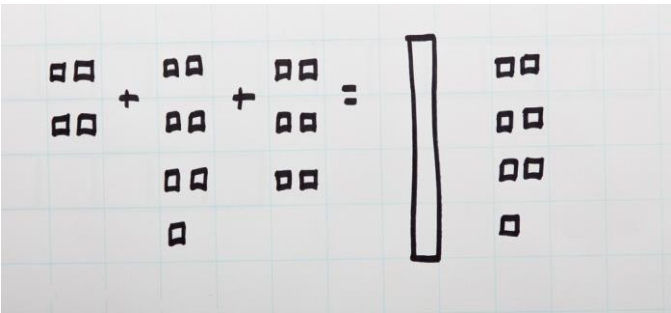
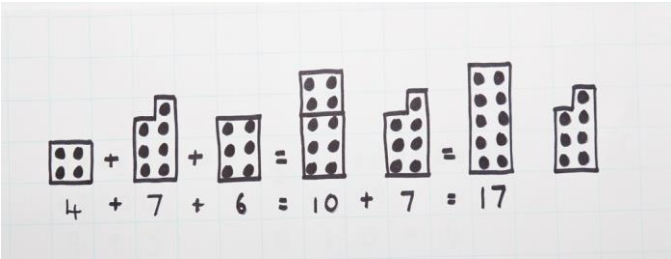


Stage 4: Adding three single digits

number bonds  
number facts  
hundreds  
tens  
ones



Start by making 10 where a pair of number bonds are included then add the third digit.



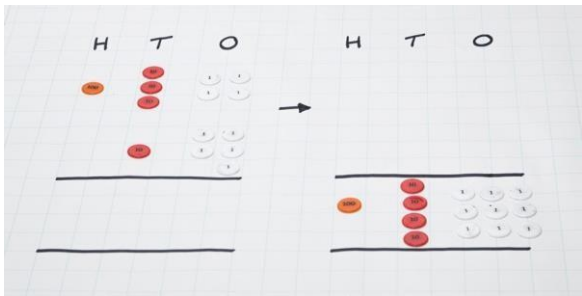
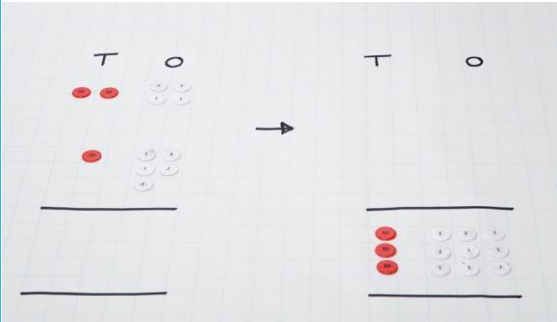
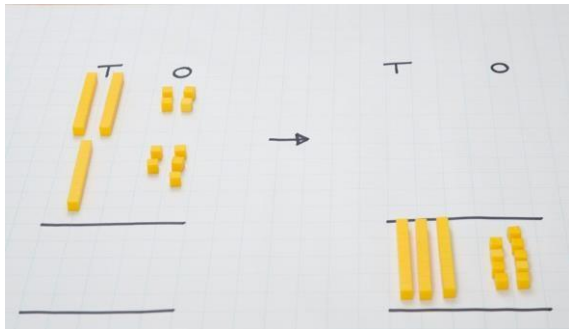
# Addition

## Objective

Stage 5: Column method - without regrouping

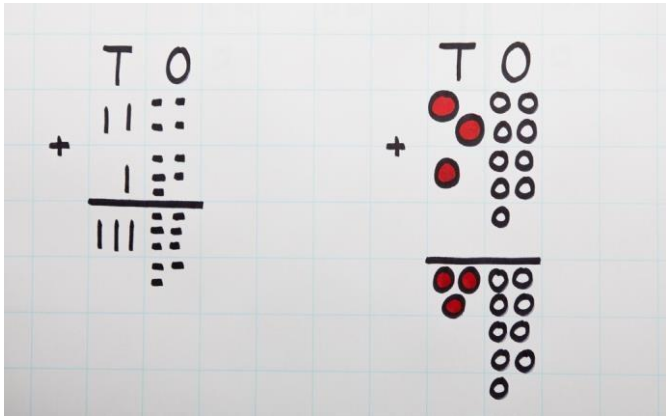
Place value column compact partition

## Concrete



Start with the bigger number and use the smaller number to make a 10 first.  
Start with the bigger number and use the smaller number to make a 10 first .

## Pictorial



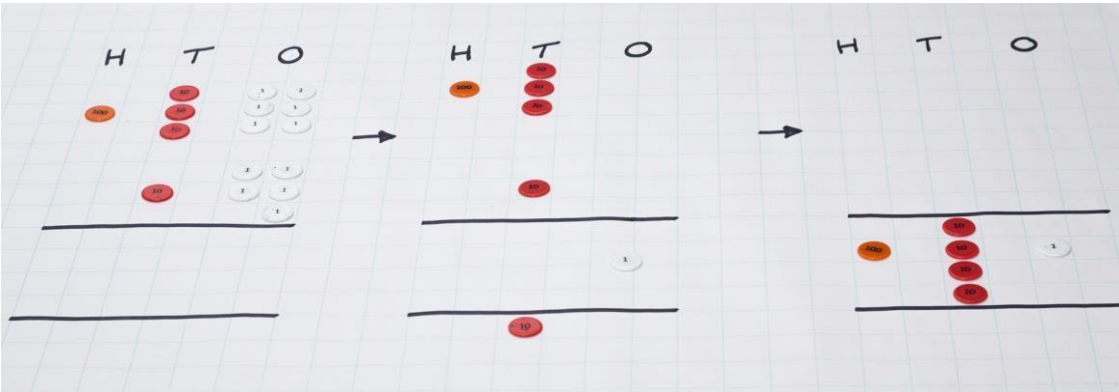
## Abstract

$$24 + 15 = 39$$

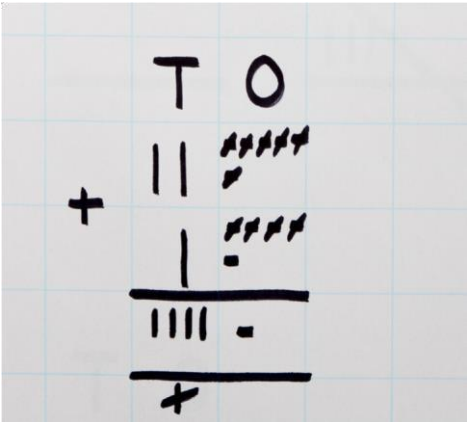
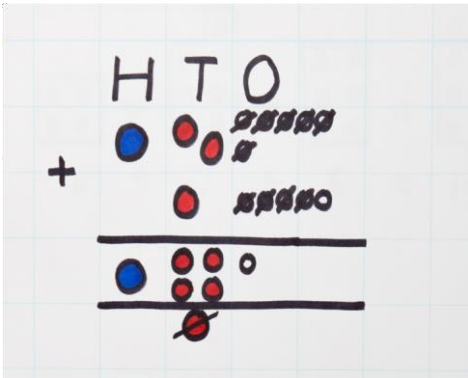
T	O
2	4
+	1 5
<hr/>	
3	9

Stage 6: Column method with regrouping

re-grouping column formal method partition



Add the ones first then the tens. Start with Base 10 before moving onto place value counters.



$$126 + 15 = 141$$

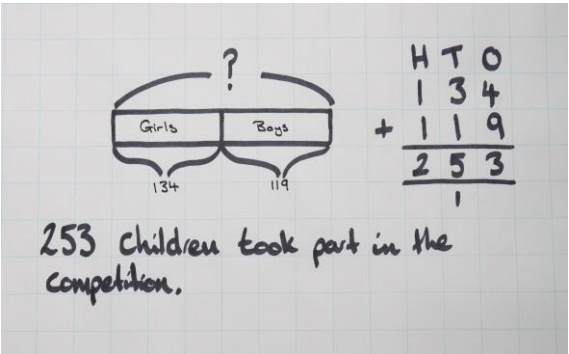
H	T	O
1	2	6
+	1	2 6
<hr/>		
1	4	1

# Addition - Bar Modelling

## Part-Part-Whole Model

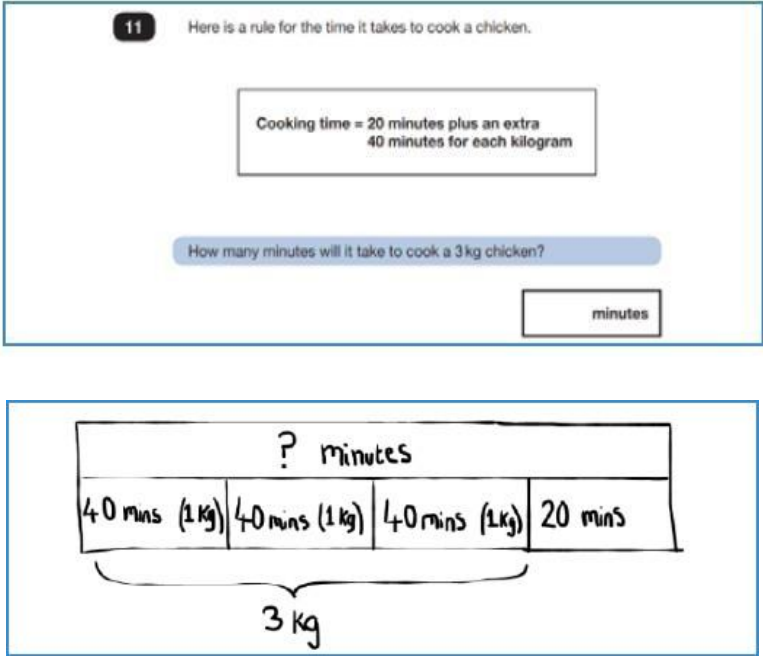
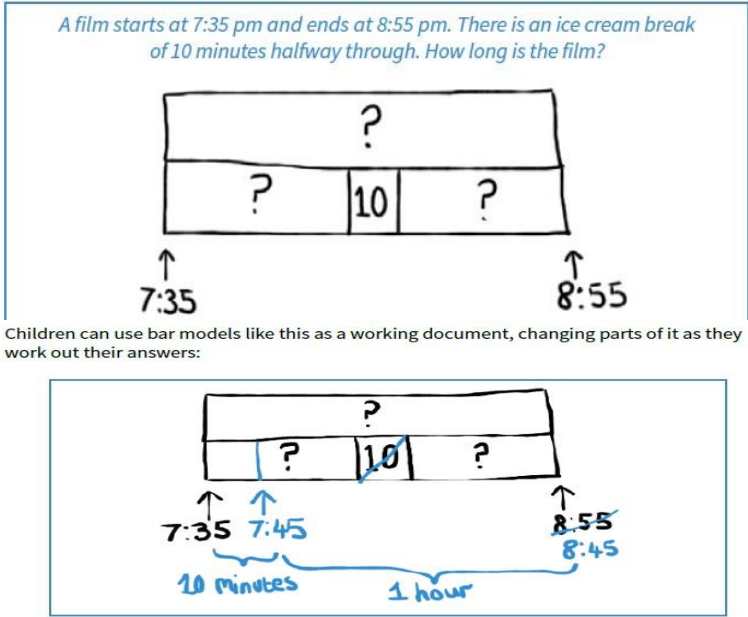
134 girls and 119 boys took part in an art competition.  
How many children took part in the competition?

We know the 2 parts. To find the whole, we add  $134 + 119$ .



### Time problems

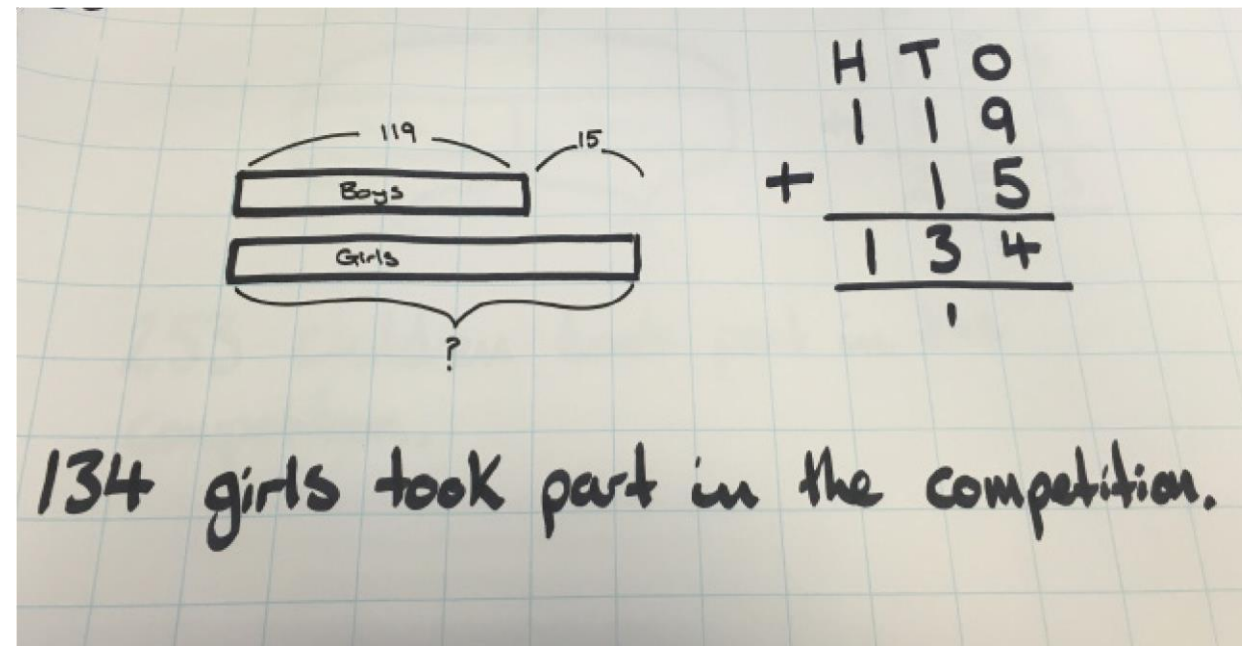
Time can be a tricky thing for children to visualise as their go-to pictorial model is a clock which goes round and round. As far as we know time doesn't go round, it moves forward in a line so bar models can represent some time problems quite well. Here's an example:



## Comparison Model

119 boys took part in an art competition. 15 more girls than boys took part. How many girls took part in the competition?

We are comparing the boys to the girls. We know the smaller quantity. To find the bigger quantity we add  $119 + 15$





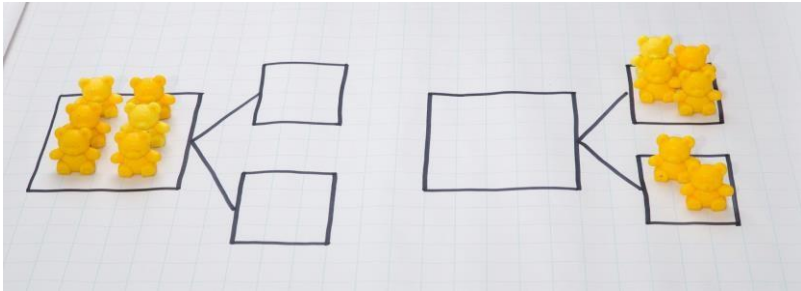
# Subtraction

## Objective

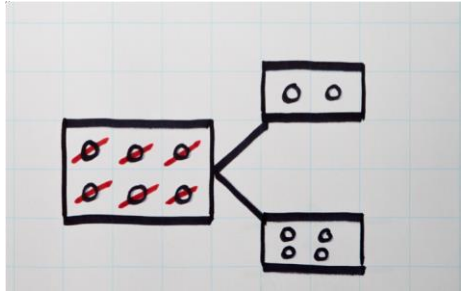
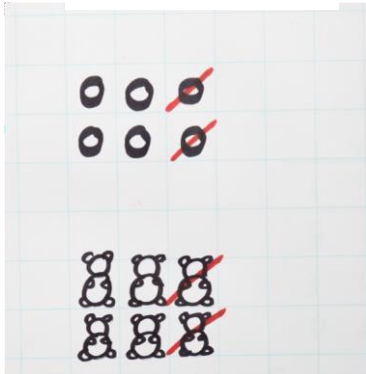
Stage 1: Taking away ones

subtract  
take away  
left over  
less

## Concrete



## Pictorial



Cross out drawn objects to show what has been taken away.

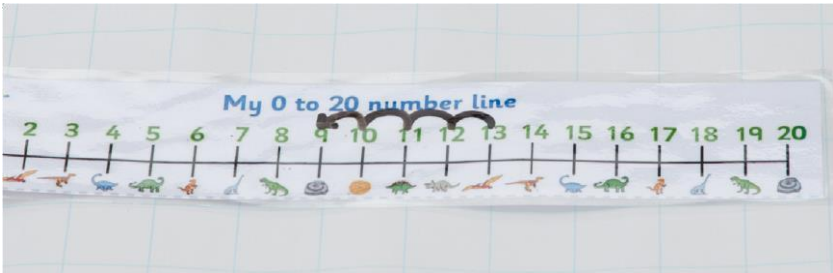
## Abstract

$$6 - 2 = 4$$

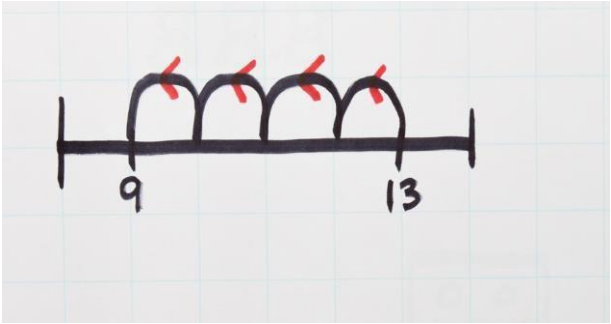
Stage 2:  
Counting Backwards

minus

tens  
ones  
fewer than  
backwards  
larger  
smaller



Make the larger number then move backwards as you count in ones.



$$13 - 4 = 9$$



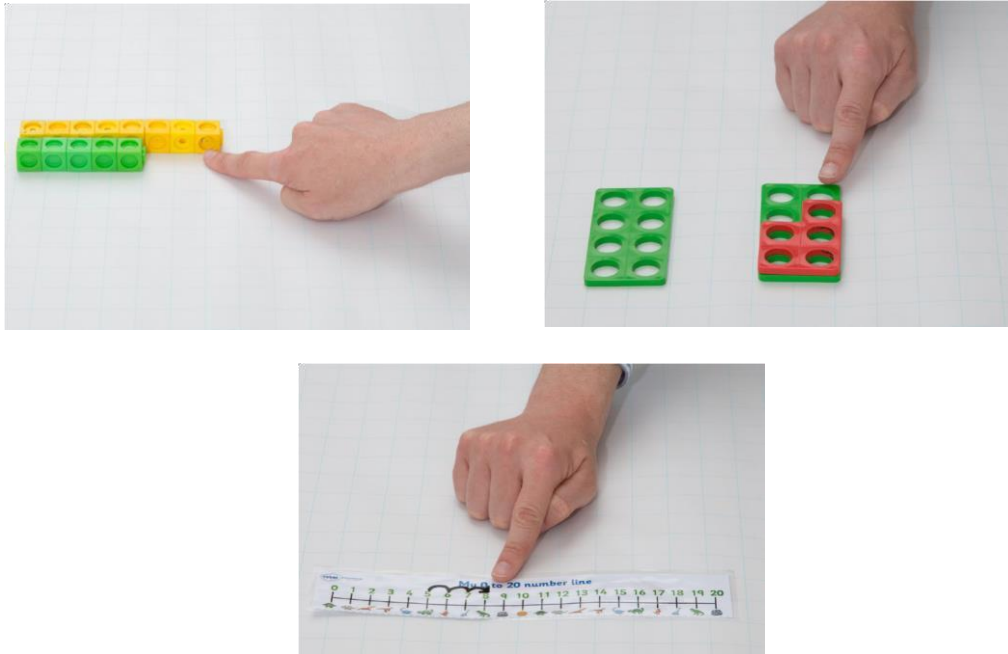
# Subtraction

## Objective

Stage 3: Finding the difference

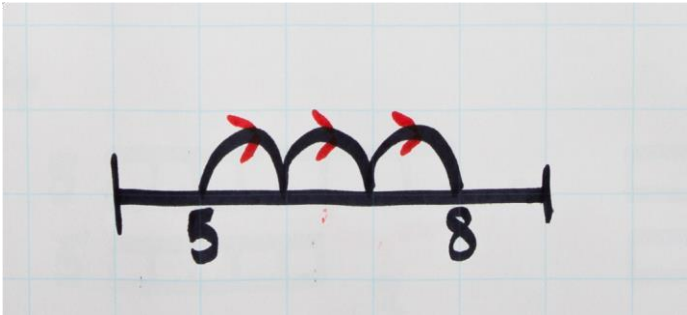
difference  
decrease  
less than

## Concrete



Finding the difference is a comparison. Pupils should be taught to compare the two numbers to find the difference.

## Pictorial



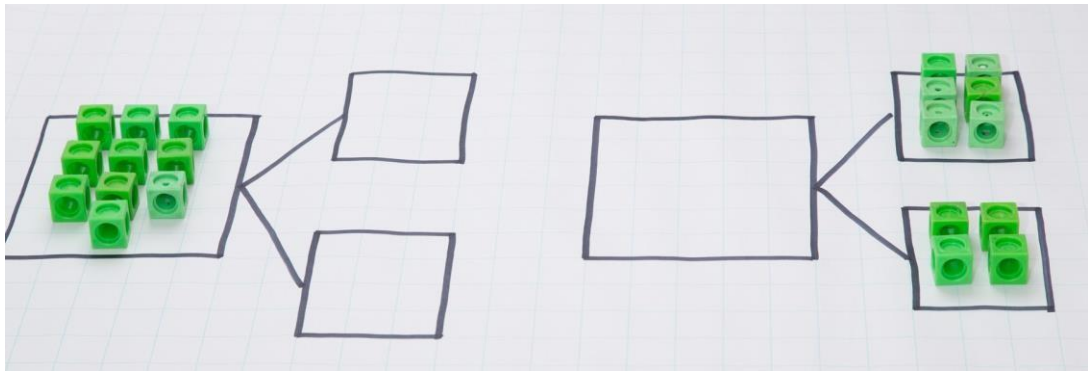
Pupils can also draw cubes to show the difference.

## Abstract

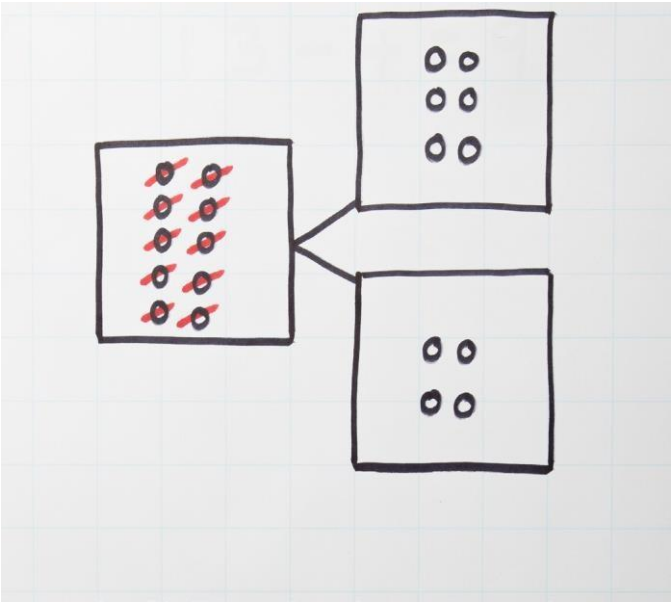
$$8 - 5 = 3$$

Stage 4: Part Part  
Whole model

inverse  
number bonds  
number facts



Link to addition to help explain the inverse.



Cross out drawn amounts to show subtraction.

$$10 - 4 = 6$$
$$10 - 6 = 4$$

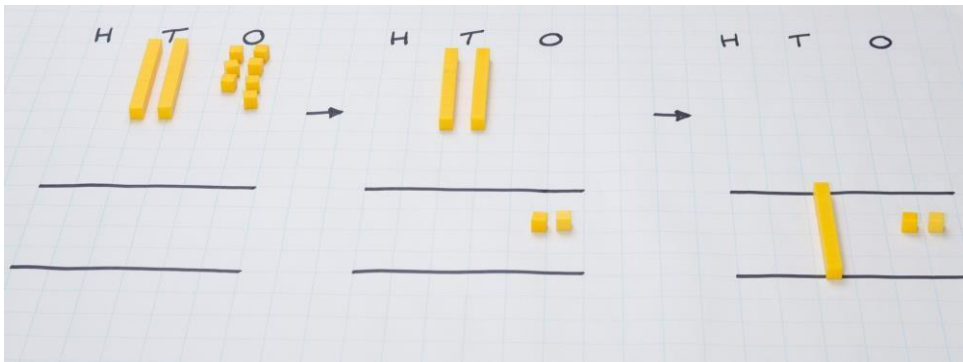
# Subtraction

## Objective

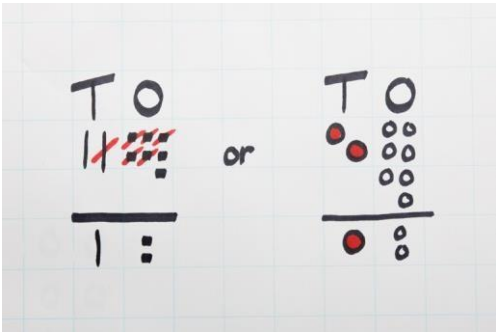
Stage 5: Column method - without regrouping

hundreds  
tens  
ones  
partition  
palce value  
column

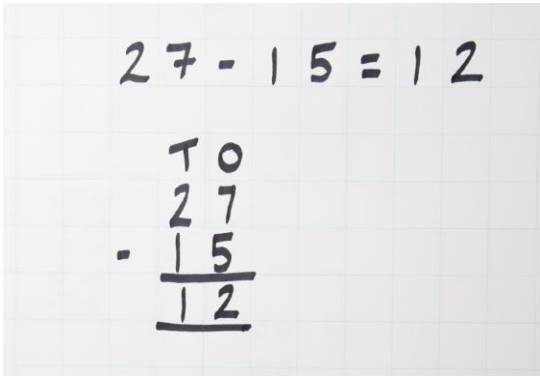
## Concrete



## Pictorial

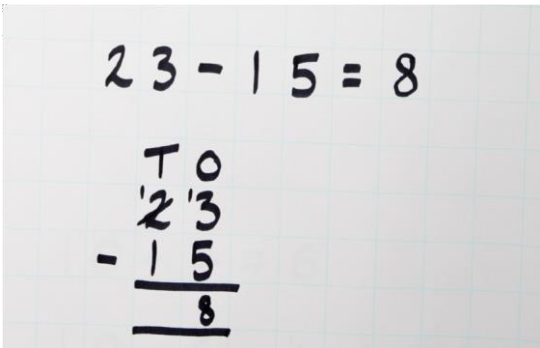
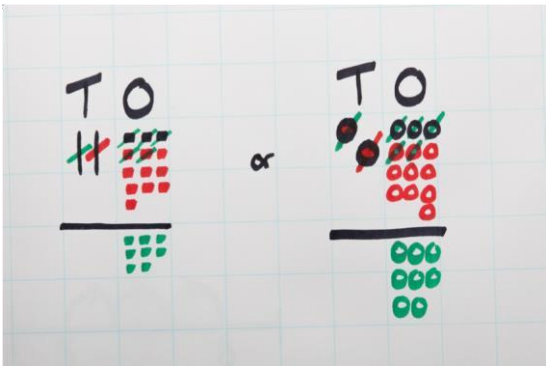
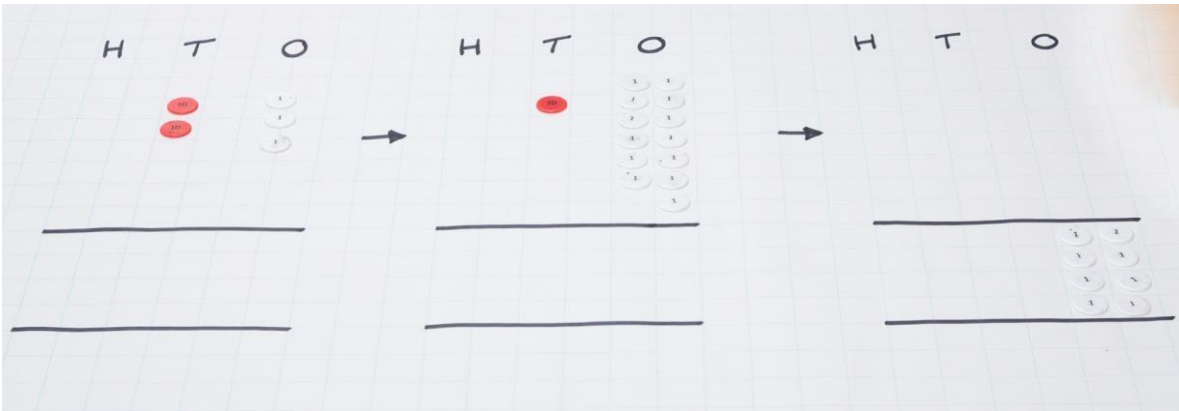


## Abstract



Stage 6: Column method with regrouping

re-grouping  
column  
partition

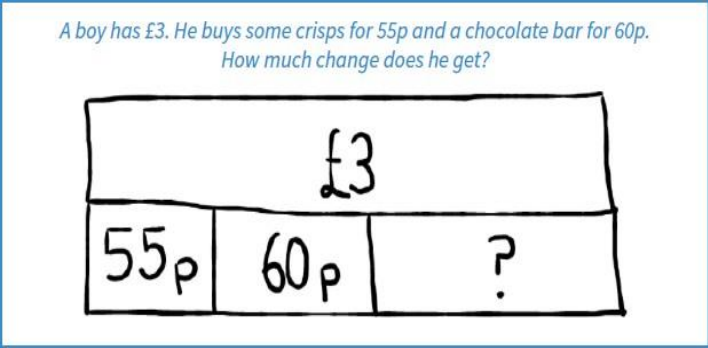
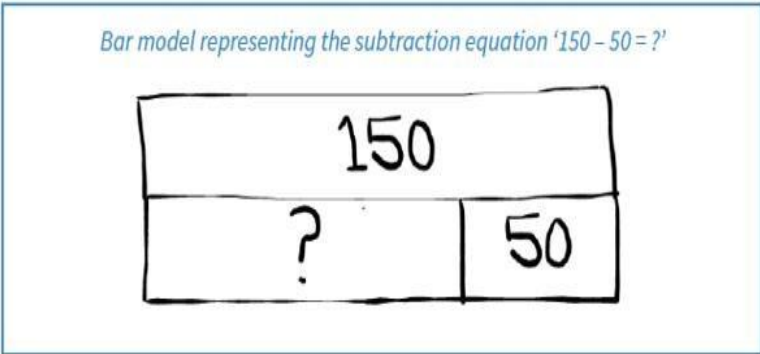
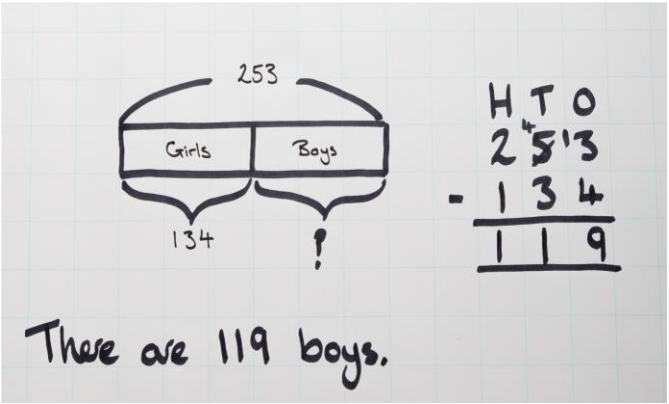


# Subtraction - Bar Modelling

## Part-Part-Whole Model

253 children took part in an art competition. There are 134 girls. How many boys are there?

We know the whole and 1 part.  
To find the missing part, we subtract  $253 - 134$ .

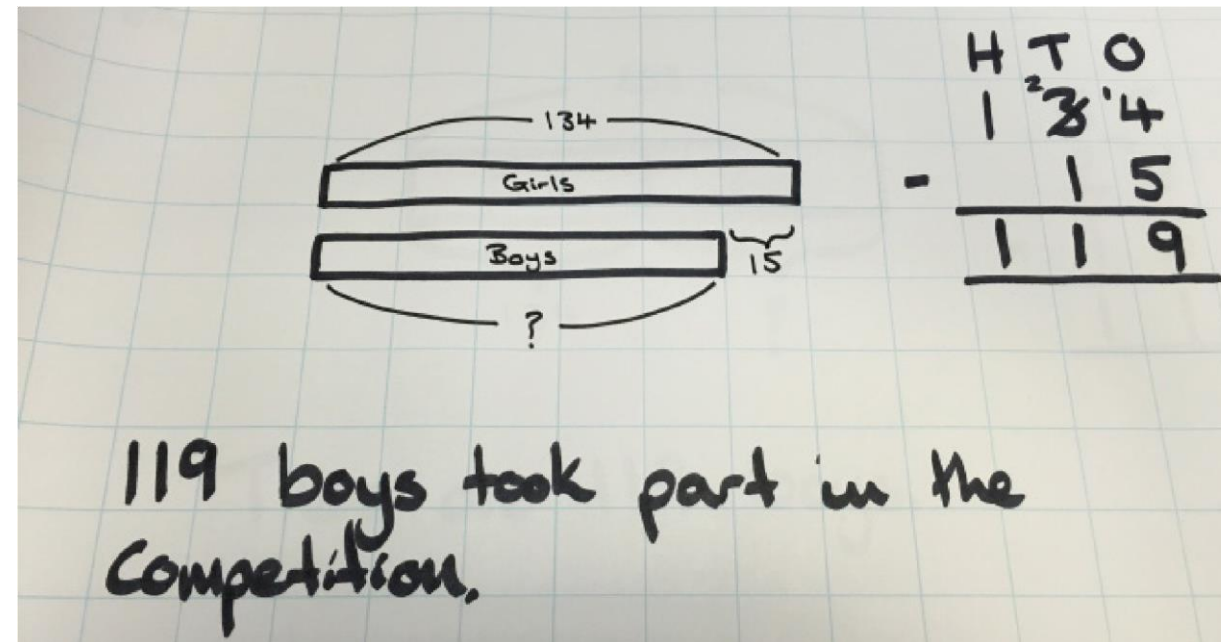




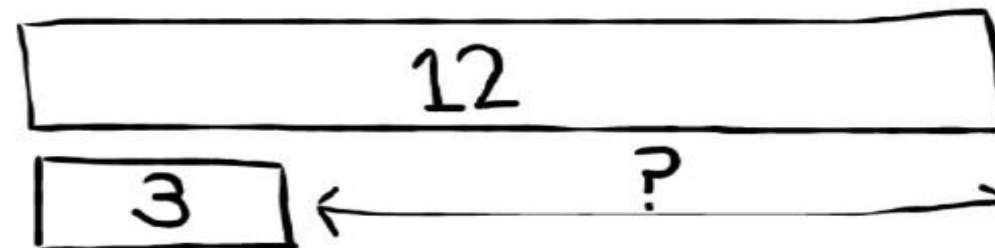
## Comparison Model

134 girls took part in an art competition. 15 fewer boys than girls took part. How many boys took part in the competition?

We are comparing the girls to the boys. We know the bigger quantity. To find the smaller quantity we subtract  $134 - 15$ .



*Sandi has 12 football cards and Umar has 3. How many more cards does Sandi have than Umar*

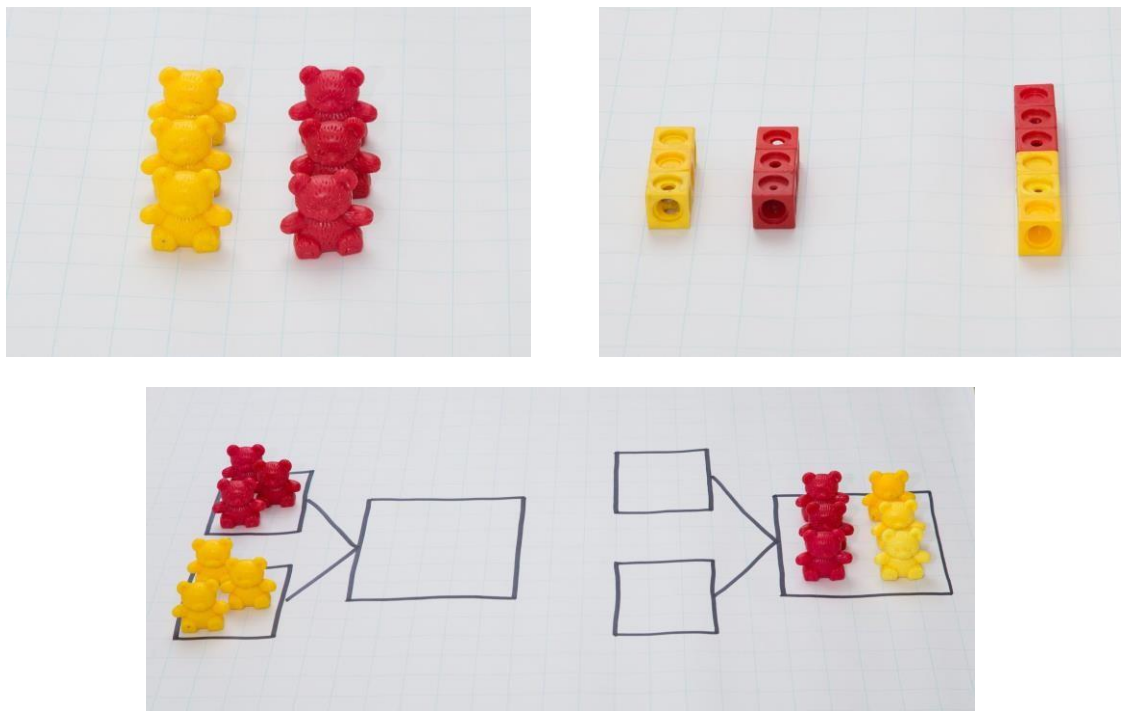


Objective

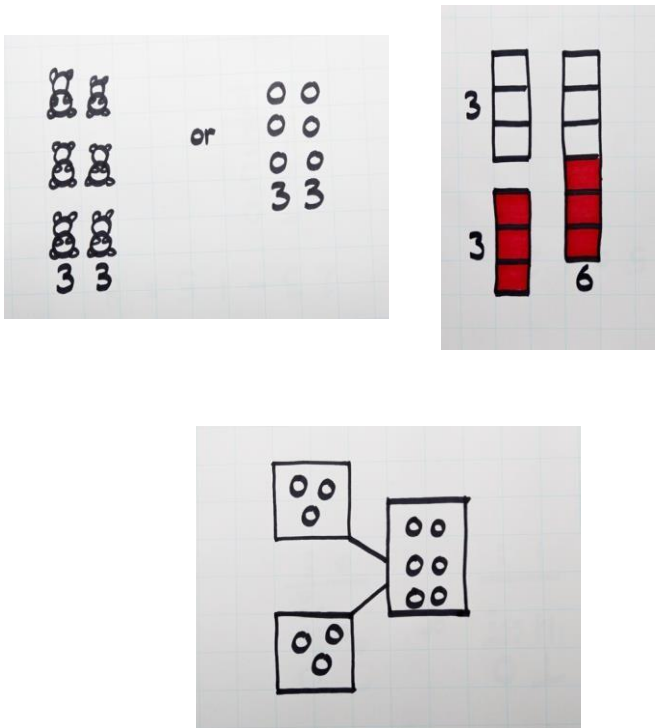
Stage 1: Doubling

double  
goups of  
total

Concrete



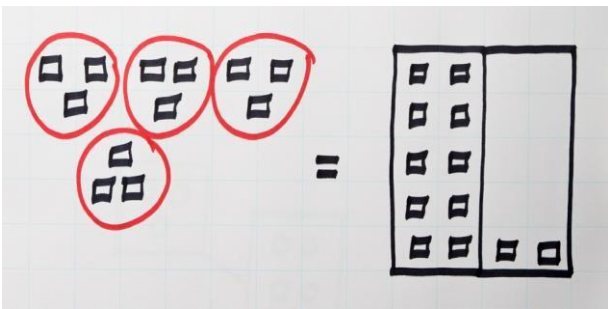
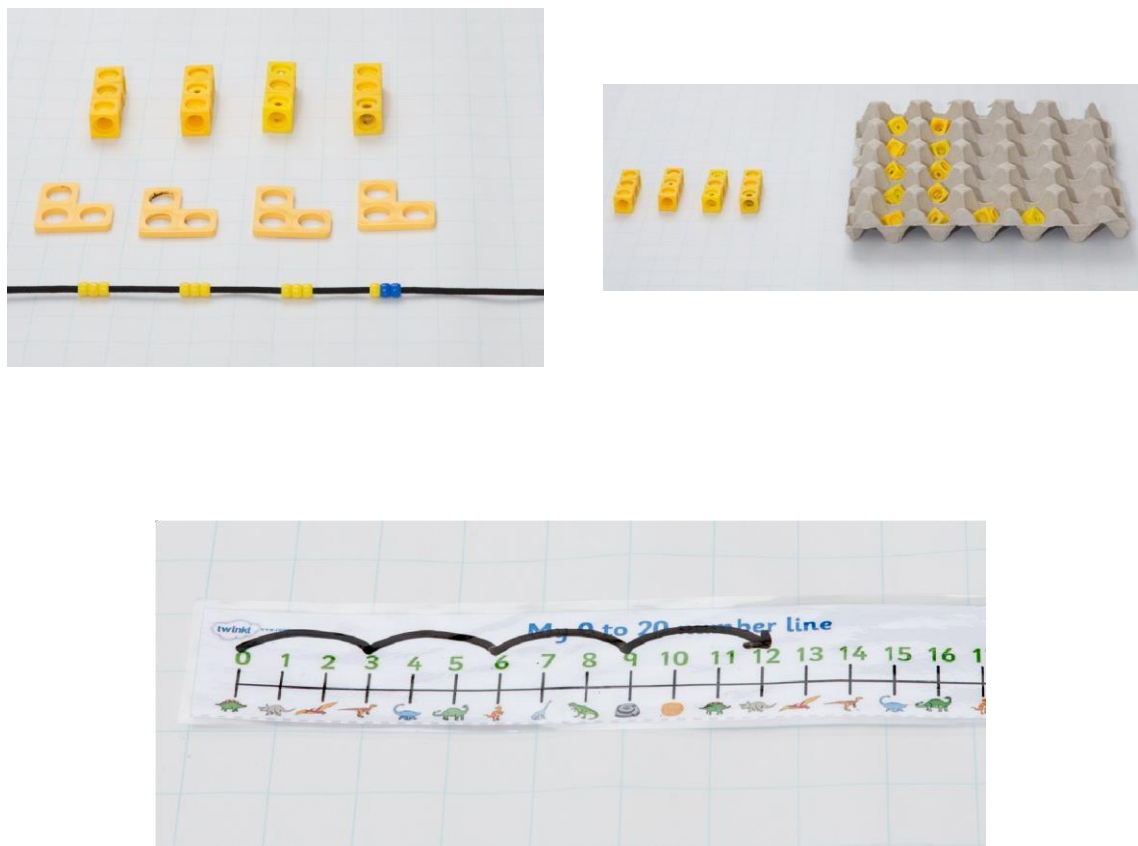
Pictorial



Abstract

Double 3 is 6  
 $2 \times 3 = 6$

Stage 2:  
Repeated addition  
then counting in  
multiples



$3 + 3 + 3 + 3 = 12$   
 $4 \times 3 = 12$

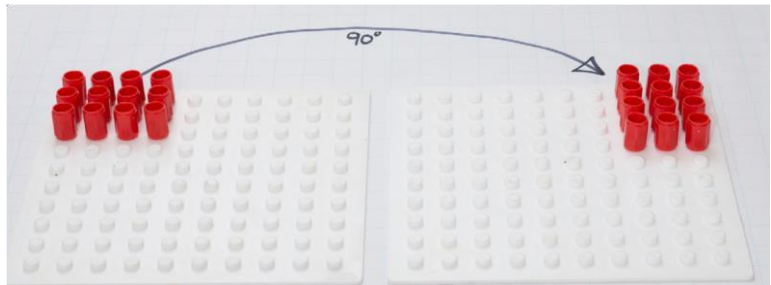
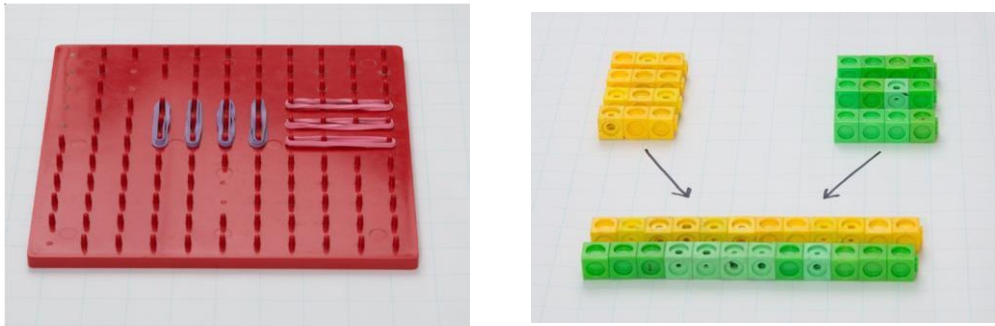
Pupils need to see repeated addition in a range of resources.

Objective

Stage 3: Arrays -  
showing  
commutative law

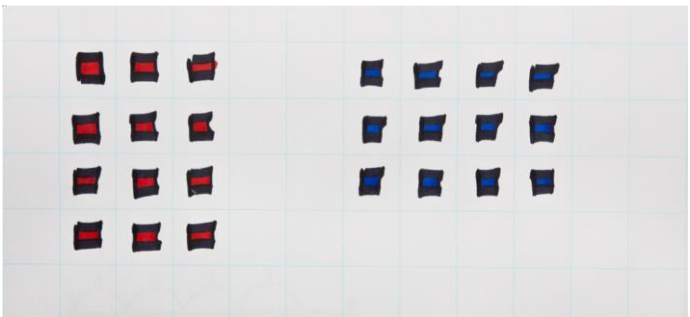
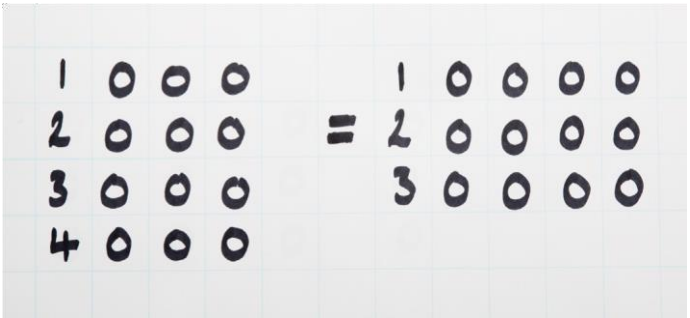
array  
commutative law  
times  
multiply  
row  
column

Concrete



Demonstrate the commutative law by making an array and turning it 90° as in the example shown above where 3 rows of 4 becomes 4 rows of three after the quarter turn.

Pictorial

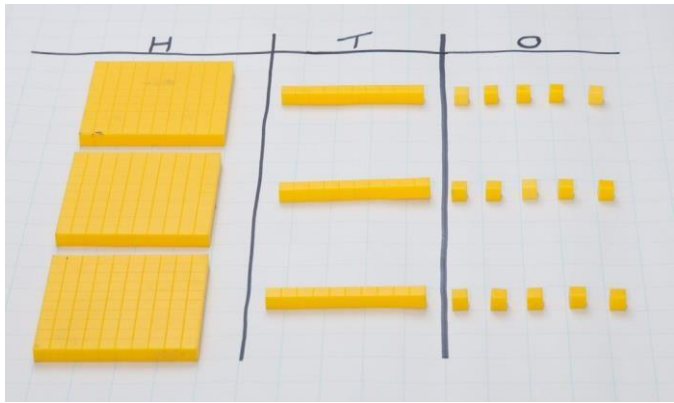
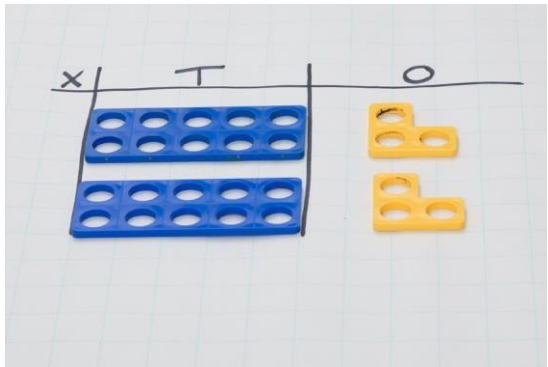
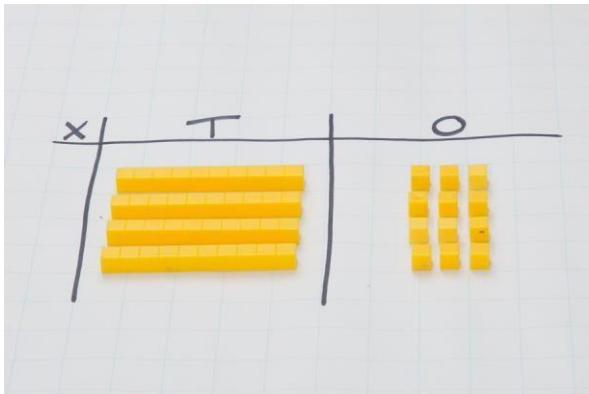


Abstract

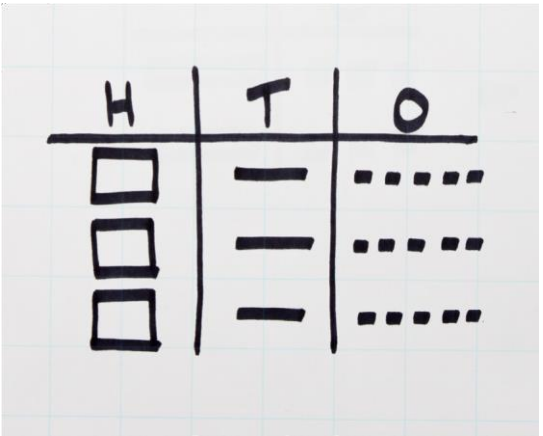
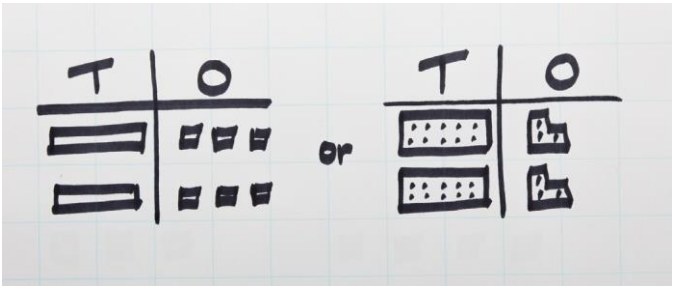
$$\begin{array}{r} 4 \times 3 = 12 \\ 3 \times 4 = 12 \end{array}$$

Stage 4: Column  
Method

column  
lots  
groups  
product  
array  
hundreds  
tens  
ones



By representing tens horizontally it makes the group clearer and keeps the calculation more compact.



$$\begin{array}{r} T O \\ 13 \\ \times 4 \\ \hline 52 \\ \hline \end{array}$$

$$\begin{array}{r} H T O \\ 115 \\ \times 3 \\ \hline 345 \\ \hline \end{array}$$





Objective

Stage 4: Column Method

- partition
- expanded
- compact efficient
- formal

Concrete

Pictorial

Once pupils are secure and move to digit X 2 digit they should just be using the formal abstract method:

Expanded method:

Handwritten expanded method for  $42 \times 74$  on grid paper. The calculation is shown in columns, with each step labeled to the right:

$$\begin{array}{r} \phantom{00}74 \\ \times \phantom{00}42 \\ \hline \phantom{00}12 \quad (3 \times 4) \\ \phantom{0}210 \quad (3 \times 70) \\ \phantom{0}240 \quad (60 \times 4) \\ 4200 \quad (60 \times 70) \\ \hline 4662 \end{array}$$

Compact (efficient) method:

Handwritten compact method for  $4276 \times 34$  on grid paper. The calculation is shown in columns, with each step labeled to the right:

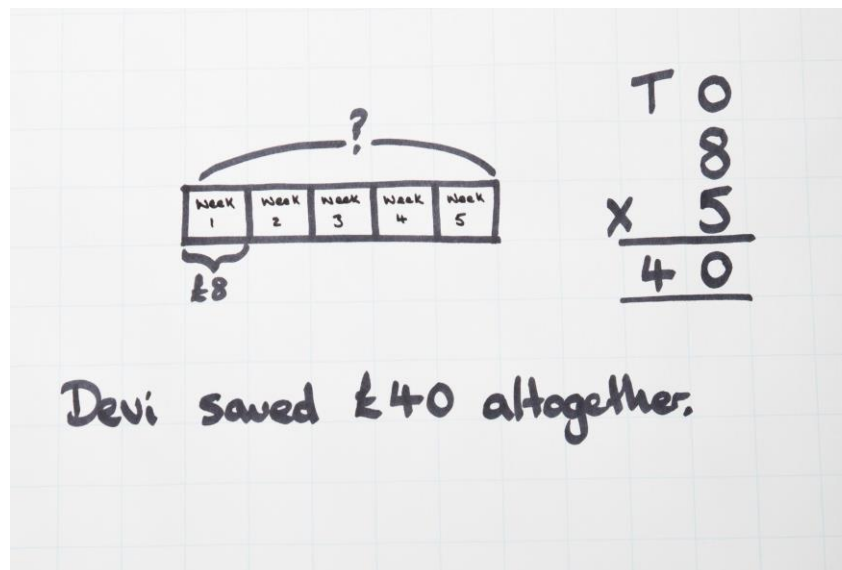
$$\begin{array}{r} \phantom{0000}4276 \\ \times \phantom{0000}34 \\ \hline \phantom{0000}17104 \\ \phantom{000}128280 \\ \hline \phantom{000}145384 \\ \phantom{0000}1 \end{array}$$

# Multiplication - Bar Modelling

## Part-Part-Whole Model

Devi saved £8 a week for 5 weeks.  
How much did she save altogether?

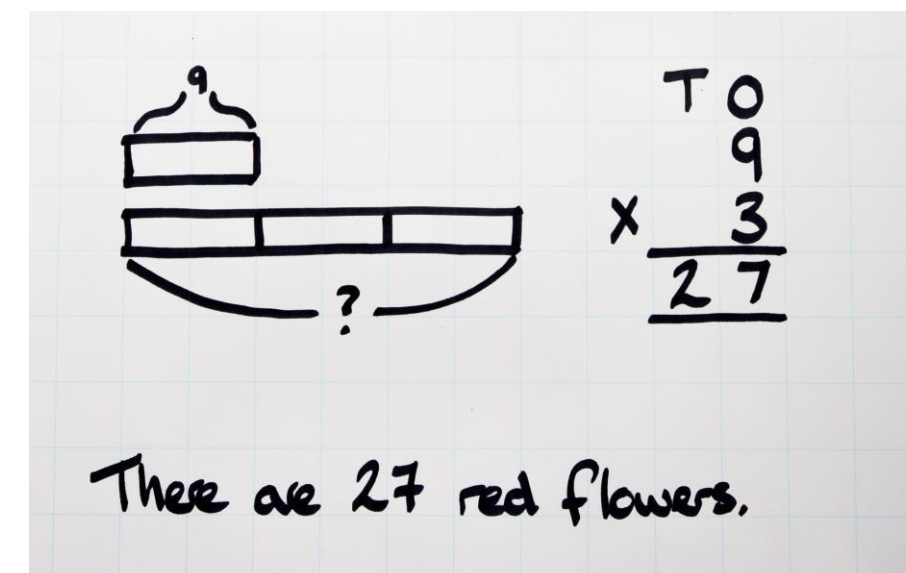
We know 1 part and the number of parts. To find the whole we multiply  $8 \times 5$ .



## Comparison Model

There are 9 white flowers. There are 3 times as many red flowers as white flowers.  
How many red flowers are there?

Two quantities are compared. One is a multiple of the other.  
We know the smaller quantity.  
To find the bigger quantity we multiply  $9 \times 3$ .



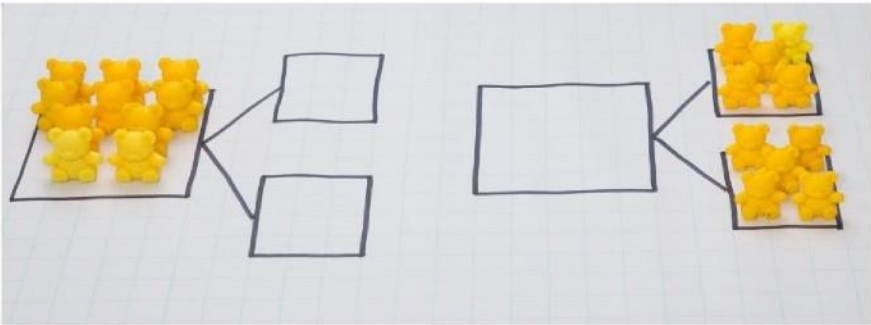
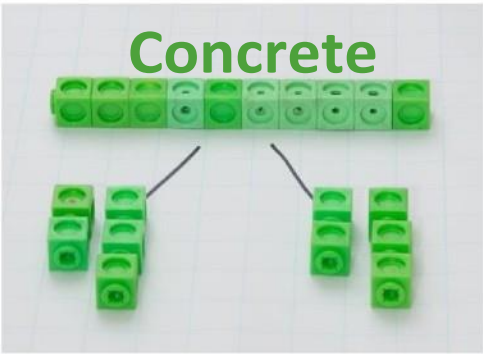




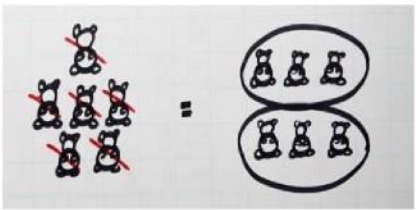
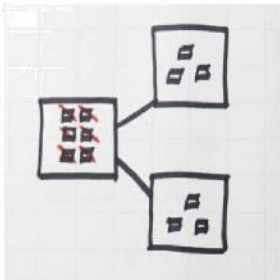
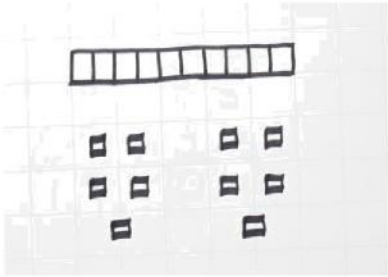
# Objective

Stage 1:  
Sharing objects into  
two groups: halving

share  
halve  
equal  
sharing into groups



## Pictorial



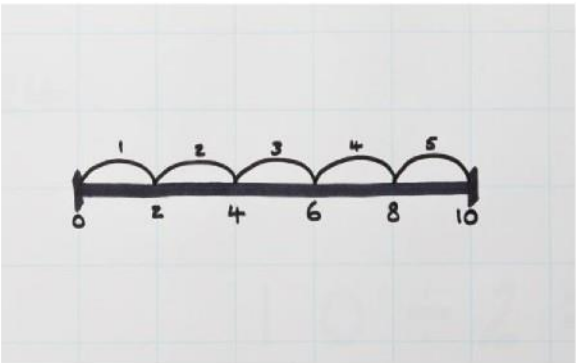
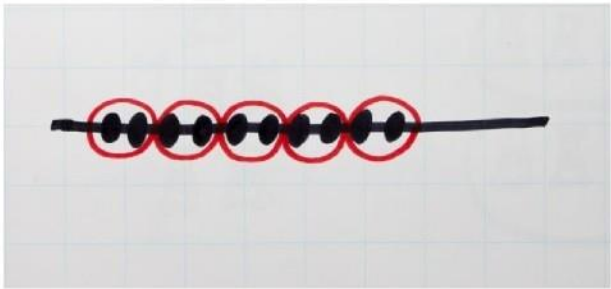
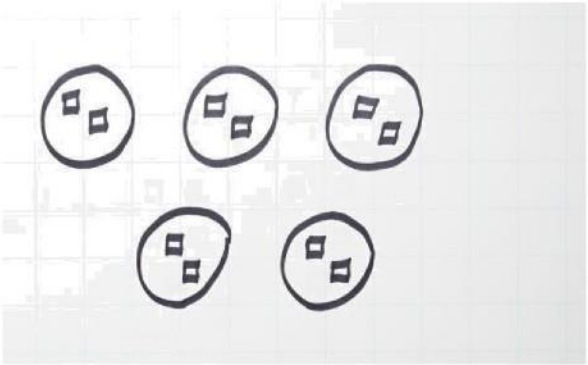
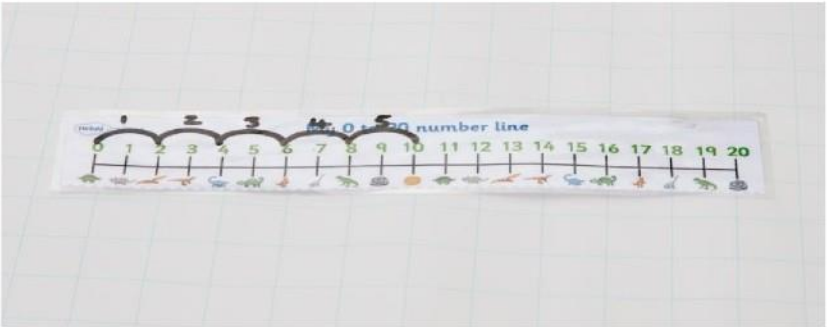
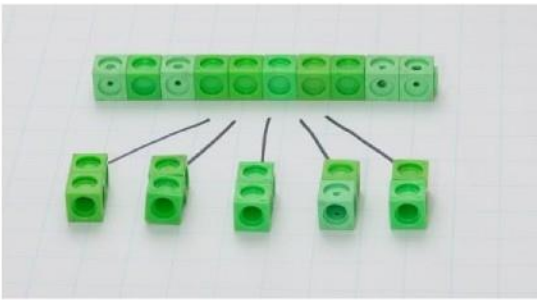
## Abstract

$$10 \div 2 = 5$$

$$6 \div 2 = 3$$

Stage 2:  
Division as grouping

sharing  
equal  
sharing into groups  
divide  
number line



$$10 \div 2 = 5$$

Divide quantities into equal groups then count the number of groups to find the answer.

# Division

## Objective

Stage 3: Arrays —  
showing Division

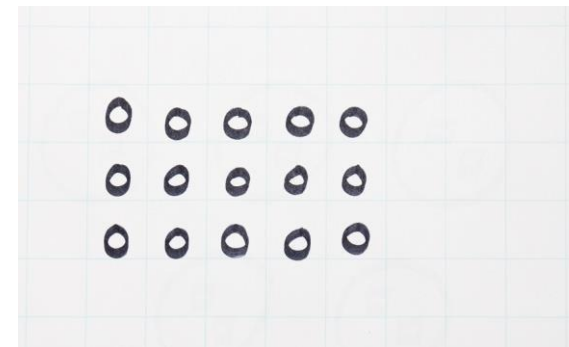
arrays  
fairly  
share equally  
inverse  
division

## Concrete



Link to multiplication

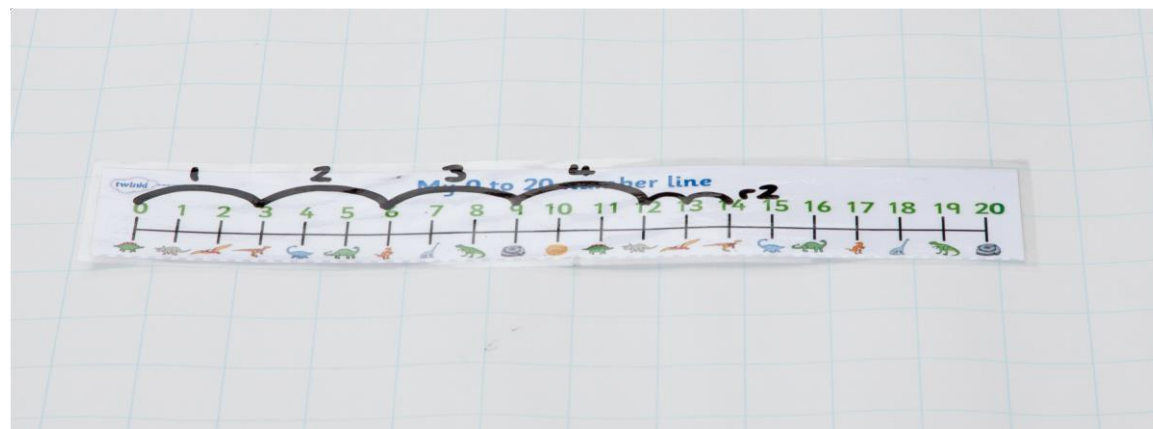
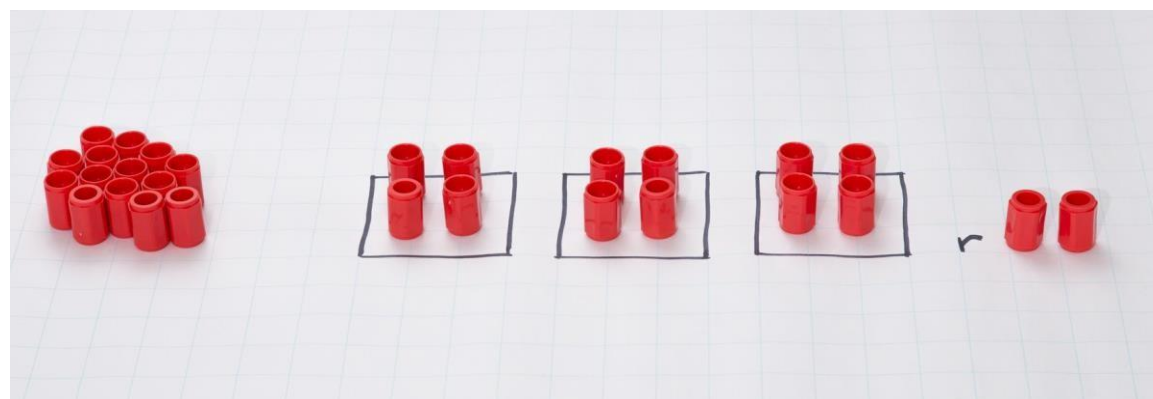
## Pictorial



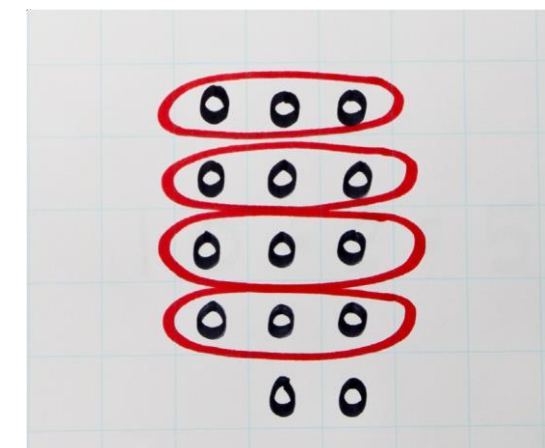
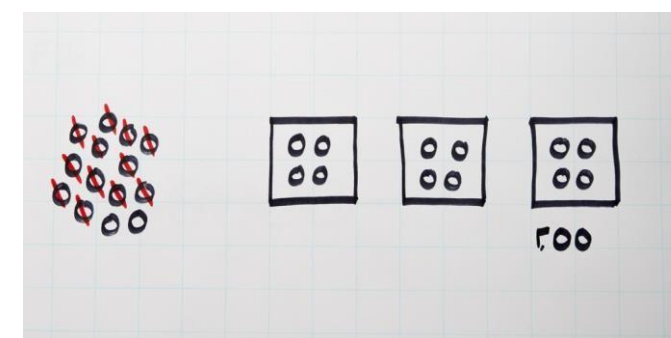
## Abstract

$$\begin{array}{l} | \ 5 \div 3 = 5 \\ | \ 5 \div 3 = 3 \end{array}$$

Stage 4: Division  
with remainders



Can also be shown with an array.



$$1 \ 4 \div 3 = 4 \text{ r } 2$$



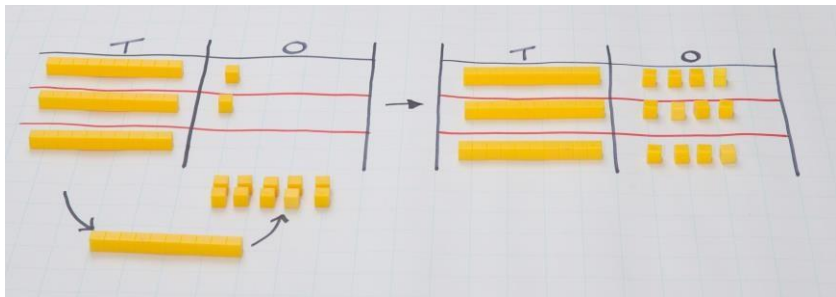
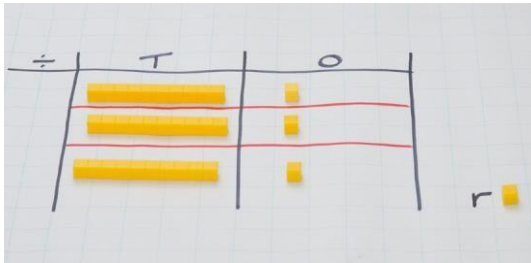
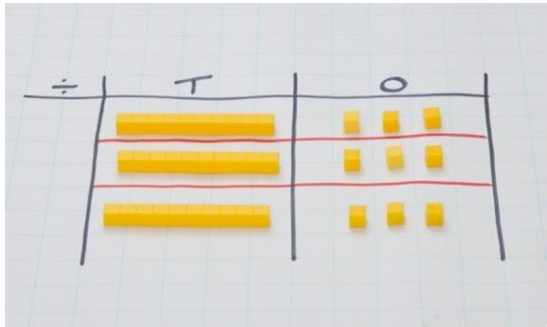
# Division

## Objective

Stage 5: Short division

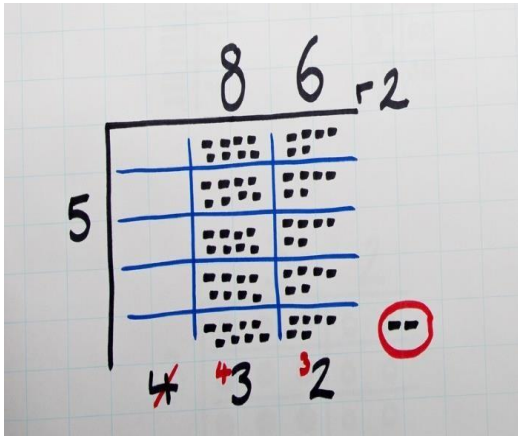
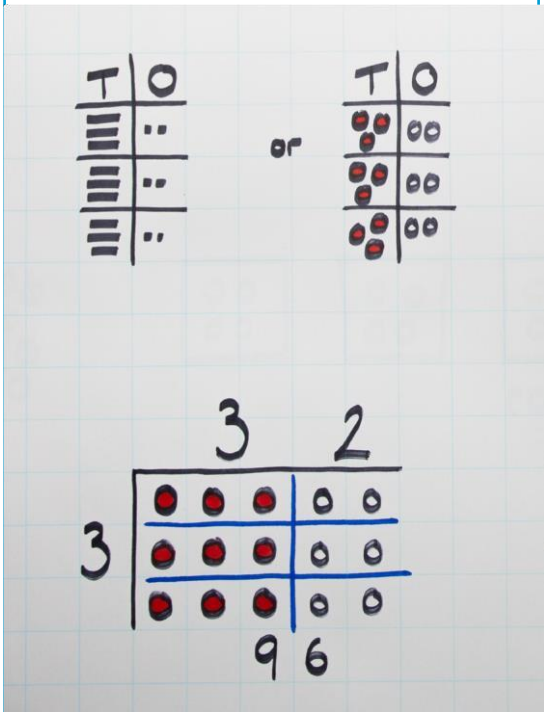
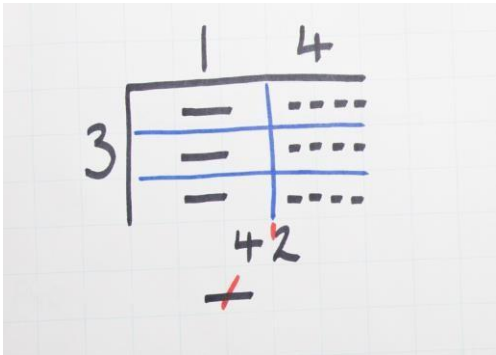
remainders  
factor  
distributive law  
associative law  
partition  
quotient  
divisible by

### Concrete



First with  
remainders then  
with re-grouping

### Pictorial



### Abstract

$$\begin{array}{r} 14 \\ 3 \overline{) 42} \\ \underline{42} \\ 0 \end{array}$$
$$42 \div 3 = 14$$

$$\begin{array}{r} 32 \\ 3 \overline{) 96} \\ \underline{96} \\ 0 \end{array}$$
$$96 \div 3 = 32$$

$$\begin{array}{r} 86 \text{ r} 2 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$



Objective

Concrete

Pictorial

Abstract

Stage 6: Long  
division

remainders  
grid method  
divisible by  
factor  
place value

A photograph of a handwritten long division problem on a light blue grid background. The problem is  $21 \overline{) 4536}$ . The quotient  $21$  is written above the dividend. The first step shows  $43$  under the first two digits of the dividend, with a remainder of  $2$ . The second step shows  $16$  under the next two digits, with a remainder of  $0$ . A blue arrow points down from the  $6$  in the quotient to the  $6$  in the dividend.

$$\begin{array}{r} 21 \\ \overline{21 \overline{) 4536}} \\ \underline{43} \phantom{00} \\ 216 \\ \underline{216} \\ 0 \end{array}$$

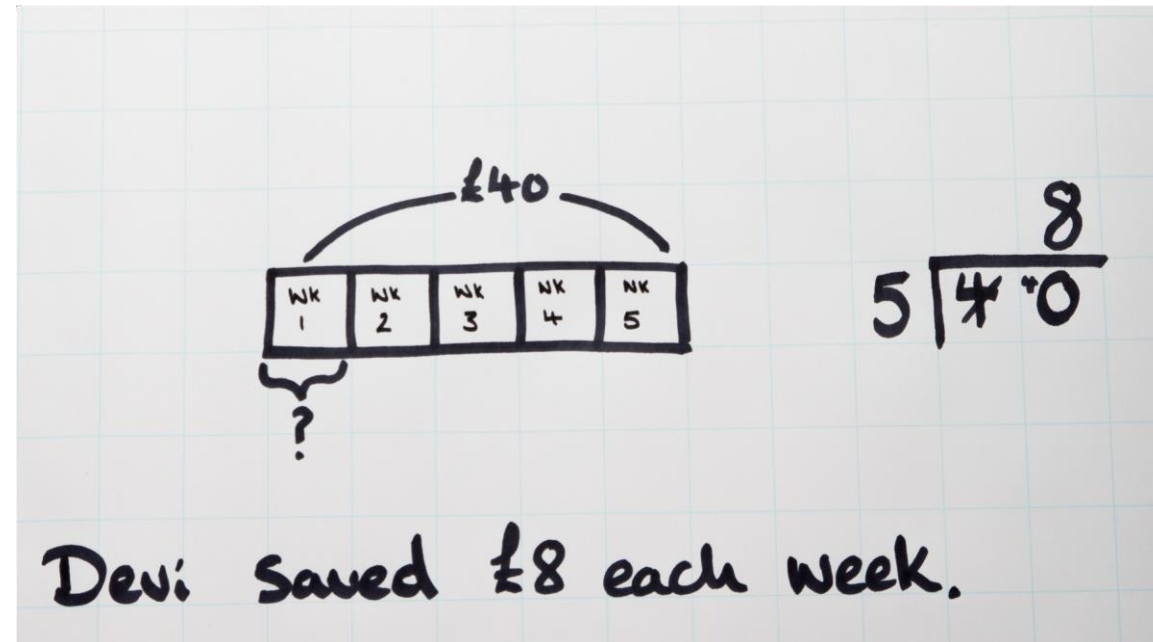
Once pupils are  
secure they can  
move to the abstract  
only long division.

# Division - Bar Modelling

## Part-Part-Whole Model

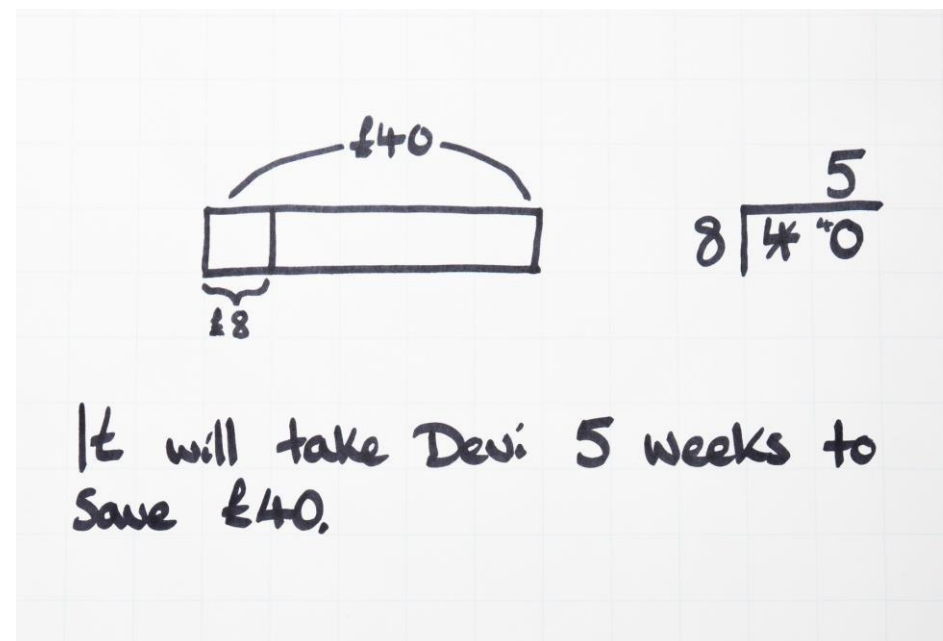
Devi saved £40 in 5 weeks.  
How much did she save each week?

We know the whole and the number of parts. To find one part we divide  $40 \div 5$ .



Devi saves £8 each week, How many weeks will it take her to save £40?

We know the whole and one part. To find the number of parts we divide  $40 \div 8$ .



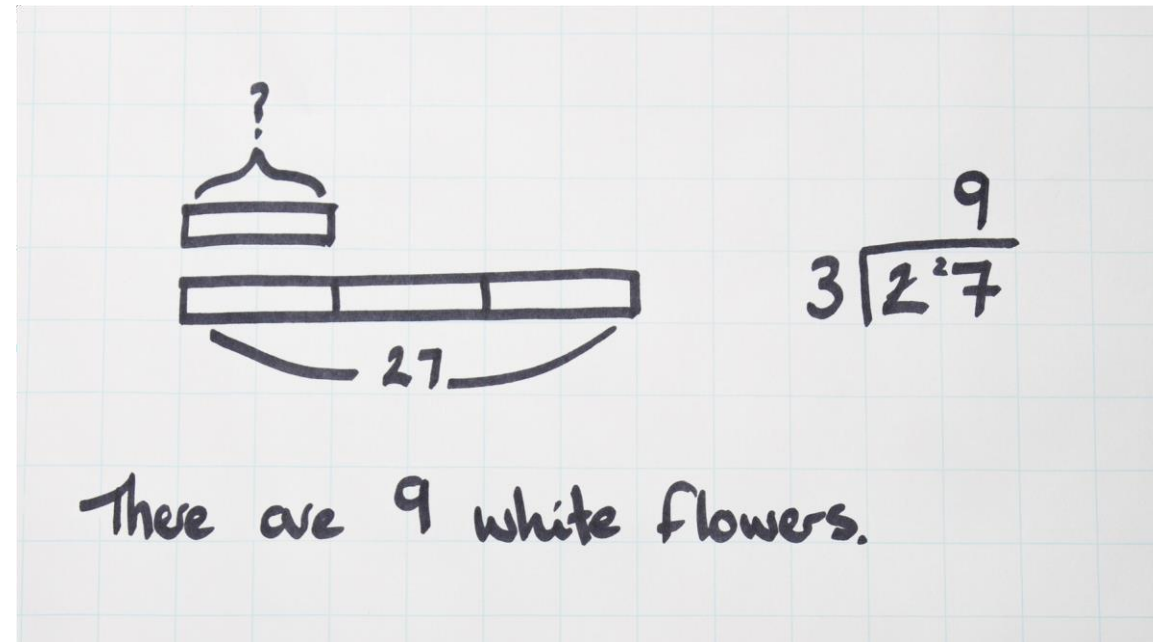


# Division - Bar Modelling

## Comparison Model

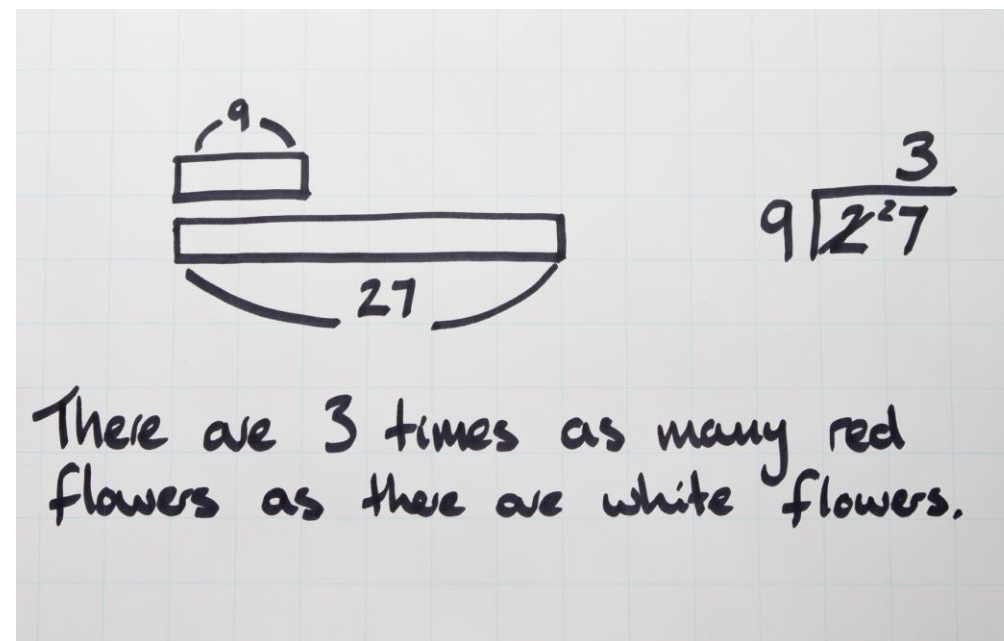
There are 27 red flowers. There are 3 times as many red flowers as white flowers. How many white flowers are there?

Two quantities are compared.  
One is a multiple of the other.  
We know the bigger quantity.  
To find the smaller quantity we divide  $27 \div 3$ .



There are 27 red flowers and 9 white flowers. How many times as many red flowers as white flowers are there?

Two quantities are compared.  
One is a multiple of the other.  
We know both quantities. To find the multiplier we divide  $27 \div 9$ .



# Fractions - Bar Modelling

## Fractions problems

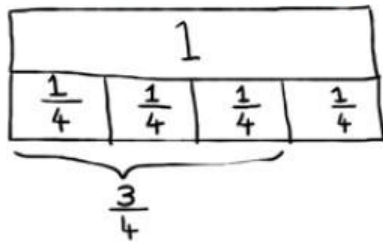
Fractions are mentioned 104 times in the National Curriculum so they're quite a big deal.

Modelling fractions using bar models is one of the most intuitive ways of showing fractions pictorially. It might even be that children will gain a greater understanding of bar modelling in general, giving them skills which are transferable to other areas of the Maths curriculum.

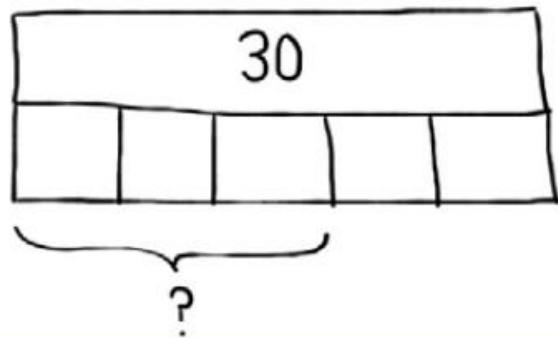
As such, there are a wide range of contexts that a bar model can be used to represent fractions problems.

In Year 2 children have to recognise, find, name and write fractions, such as  $\frac{3}{4}$  which can be represented as below:

*A bar model representing three quarters*



*A bar model representing the equation: What is  $\frac{3}{5}$  of 30?*



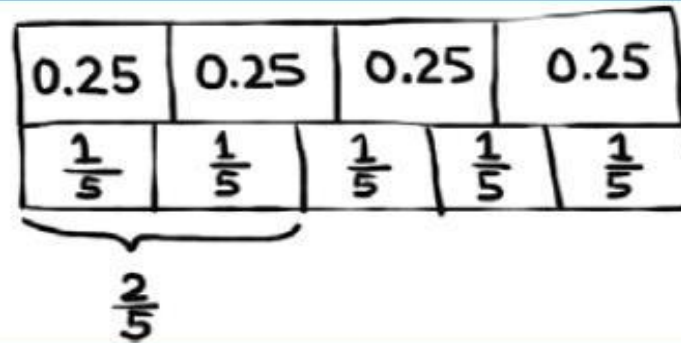
20

Adam says,

0.25 is smaller than  $\frac{2}{5}$



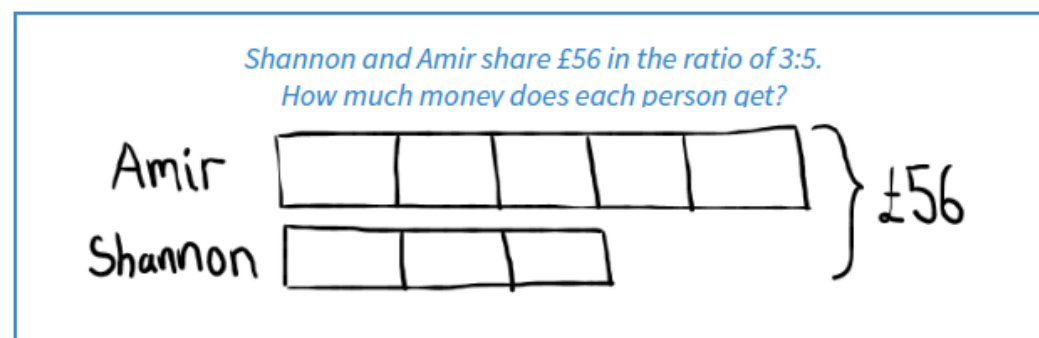
Explain why he is correct.



# Ratio and algebra - Bar Modelling

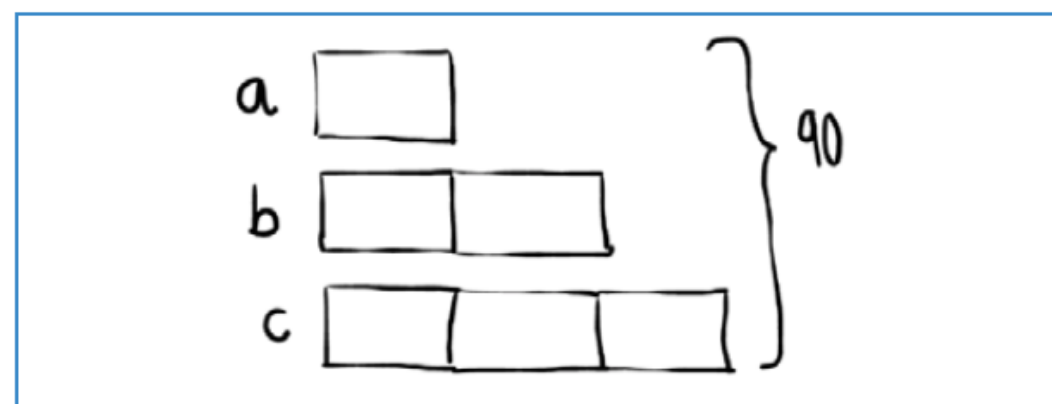
## Ratio problems

The comparison bar model is a gift when it comes to ratio problems, which is particularly significant given that in the 2017 KS2 tests the ratio question was one of the most poorly answered.



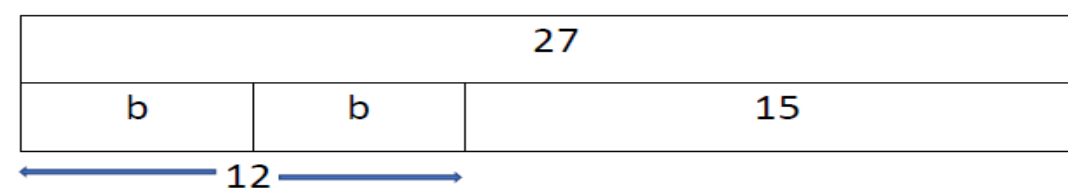
90 sweets are shared between bowls *a*, *b* and *c*. Bowl *b* contains twice the amount that bowl *a* contains. Bowl *c* contains three times the amount that bowl *a* contains. How many more sweets does bowl *b* have than bowl *a*?

Both these versions of the problem can be represented using the bar model below:



How to teach equations with the bar model:

$2b + 15 = 27$  What is the value of *b*?



From this we can see that  $27 - 15 = 2b$

So, we can see  $b = 6$

$2a + 7 = a + 11$

So, what do we know?

